A REMARK ON ELEMENTS OF $D'_{L^q}(\mathbb{R})$ AS BOUNDARY VALUES OF HOLOMORPHIC FUNCTIONS

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Abstract

In this paper we give a boundary value characterization of the space $D'_{L^p}(\mathbb{R})$. Namely, we will show that every distribution that belongs to the space $D'_{L^p}(\mathbb{R})$ can be represented by analytic functions. We will also show that analytic functions fulfilling certain growth conditions have their boundary values in the space $D'_{L^p}(\mathbb{R})$. This characterization of holomorphic function spaces whose elements have boundary values in spaces of distributions is different from the one given in [6]. By assuming a different condition on the growth of a holomorphic function we obtain characterization which does not reffer to almost analytic continuation as in [6] and improves the results of [4].

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In this note we will use the notations introduced by L. Schwartz in [5]. The norm in L^p is denoted by $\|\cdot\|_{L^p}$. $\mathbb{N}$ denotes the set of positive integers, $\mathbb{N}_0 = \mathbb{N} \cup\{0\}$.

Definition 1. We define the spaces $D_{L^q}(\mathbb{R})$ for $q \in \mathbb{N}$ as follows:

$$D_{L^q}(\mathbb{R}) = \{\phi \in C^\infty(\mathbb{R}) : \quad \|\phi^{(n)}\|_{L^q} < \infty \quad \text{for} \quad n \in \mathbb{N}_0\}$$

and introduce the norms

$$\|\varphi\|_m := \left(\sum_{n=0}^{m} \int_{\mathbb{R}} |\varphi^{(n)}(x)|^q dx\right)^{\frac{1}{q}}.$$

We say that a sequence $(\varphi_n)_{n\in\mathbb{N}}$ converges to zero in the space $D_{L^q}(\mathbb{R})$, if for each $m \in \mathbb{N}$ the sequence of norms $\|\varphi_n\|_m$ converges to zero as n tends to infinity.

Definition 2. Let $q > 1$ and let $\frac{1}{p} + \frac{1}{q} = 1$. The set of all linear functionals continuous on the space $D_{L^q}(\mathbb{R})$ will be denoted by $D'_{L^p}(\mathbb{R})$. The elements of this set are called *generalized functions of the class* $D'_{L^p}(\mathbb{R})$.